

A W -DRESSED ELECTROWEAK STRING

P. Olesen

The Niels Bohr Institute
University of Copenhagen
Blegdamsvej 17
DK-2100 Copenhagen
Denmark

Abstract

We give plausibility arguments for the existence of a W -dressed electroweak string for the Weinberg-Salam model with $\theta_W = 0$. This string is a Z -string which in the core has a finite energy contribution from W -condensation induced by the anomalous magnetic moment in the Yang-Mills field. The solution which has minimum energy at $r = 0$ interpolates between the unbroken ($r = 0$) and the broken ($r \rightarrow \infty$) $SU(2) \times U_y(1)$ phase.

Recently it has been shown by Vachaspati [1] that the electroweak Weinberg–Salam theory [2] has vortex solutions. This solution is formed from the lower component of the Higgs field and the Z field, and it satisfies the usual vortex equations of motion [3]. These electroweak strings are not stable on topological grounds, and hence their stability must be investigated. This was done [4], and stability was found for a range of parameters unfortunately not within the experimental limits. It has been suggested [5] that perhaps the inclusion of at least two Higgs multiplets may improve this situation.

It has, however, been pointed out by Perkins [6] that the electroweak string can have its energy lowered by the formation of a W –condensate in its core¹. This is because of the fact that the electroweak vacuum is unstable to the formation of a W –condensate [7] if a strong “magnetic” Z –field is applied. The physical reason for this effect is that the electroweak vacuum is “paramagnetic”, with an energy which contains the terms

$$-\cos\theta_W \mathbf{B}^Z \cdot \boldsymbol{\mu} + \boldsymbol{\mu}^2 \quad , \quad (1)$$

where $B_i^Z = \frac{1}{2}\varepsilon_{ijk}Z_{jk}$ and the magnetic moment density $\boldsymbol{\mu}$ is given by $(W_i = \sqrt{\frac{1}{2}}(W_i^1 + iW_i^2))$

$$\mu_j = ig\varepsilon_{jkl}W_k^\dagger W_l \quad (2)$$

Although the energy contains many other terms than those exhibited in eq. (1) it is clear that eq. (1) can give rise to a W –condensation under some circumstances.

The result obtained by Perkins [6] shows that it is possible to find W –configurations which lower the energy of the Z –string, provided $\sin\theta_W < 0.9$. In this connection it should be noticed that the first term in eq. (1) vanishes for $\theta_W \rightarrow \pi/2$, so for θ_W close to $\pi/2$ very little energy is gained by W –condensation. Thus, for the experimental value $\sin^2\theta_W \approx 0.23$ W –condensation should occur in the core of the Z –string.

In this paper we present plausability arguments that there exists a “ W –dressed Z –string” for small values of θ_W . In order to simplify the calculations we take $\theta_W = 0$ in the following. This should be satisfactory as far as a first orientation is concerned, since the usual Z –string is stable near $\theta_w = \pi/2$ [4], and hence the problem of instability is most marked near $\theta_W = 0$.

In order to have a solution for a W –dressed electroweak string, we need to satisfy all the equations of motion of the electroweak string. In order to produce an ansatz which can work, one has to be very careful. Let us start with the Higgs field ϕ . We would like to have only a single complex field. Let us therefore start by asking if the upper complex component ϕ_U in

$$\phi = \begin{pmatrix} \phi_U \\ \phi_L \end{pmatrix} \quad (3)$$

can be put to zero. This is not a trivial problem:² Consider the equation of motion for ϕ_U obtained from the Lagrangian by varying ϕ_U^* . This equation has the following structure:

$$(\text{terms which vanish when } \phi_U = 0) + \frac{1}{2}ig(W_i^\dagger\partial_i\phi_L + \partial_j(W_j^\dagger\phi_L)) = 0 \quad . \quad (4)$$

The term written out explicitly apparently does not vanish for $\phi_U = 0$. If this is correct, then $\phi_U = 0$ is not a solution of the equations of motion (4).

¹This point was also mentioned by the author in a communication to T. Vachaspati.

²I thank T. Vachaspati for pointing this out to me.

However, it turns out by closer inspection that the explicit term in eq. (4) does actually vanish for $\phi_U = 0$. This can be seen from the equations of motion for W_μ and Z_μ . These equations together produce an integrability condition. In order to simplify the equations, we start by taking $\phi_U = 0$, $\phi_L \neq 0$. It then turns out that the explicit term in eq. (4) vanishes because of the integrability condition, and hence it is perfectly consistent to have $\phi_U = 0$.

For the moment, consider an arbitrary θ_W . The equation for W_μ is then

$$D_\mu(D_\mu W_\nu - D_\nu W_\mu) - igF_{\mu\nu}^3 W_\mu - \frac{1}{2}g^2|\phi_L|^2 W_\nu = 0 \quad , \quad (5)$$

$$F_{\mu\nu}^3 = \partial_\mu V_\nu^3 - \partial_\nu V_\mu^3 - ig(W_\mu^\dagger W_\nu - W_\nu^\dagger W_\mu) \quad , \quad (6)$$

$$V_\mu^3 = Z_\mu \cos \theta_W + A_\mu \sin \theta_W \quad , \quad D_\mu = \partial_\mu + igV_\mu^3 \quad . \quad (7)$$

Now operate with D_ν on eq. (5), which then reduces to

$$-g^2(W_\mu^\dagger W_\nu - W_\nu^\dagger W_\mu)D_\nu W_\mu + ig \partial_\nu F_{\mu\nu}^3 W_\mu + \frac{1}{2}g^2 D_\nu(|\phi_L|^2 W_\nu) = 0 \quad . \quad (8)$$

In this expression we can use the equation for $F_{\mu\nu}^3$,

$$-\partial_\mu F_{\mu\nu}^3 + ig(W_\mu F_{\mu\nu}^\dagger - W_\mu^\dagger F_{\mu\nu}) + \frac{g^2}{2\cos\theta_W}|\phi_L|^2 Z_\nu + \frac{ig}{2}(\phi_L \partial_\nu \phi_L^* - \phi_L^* \partial_\nu \phi_L) = 0 \quad . \quad (9)$$

Using this in eq. (8) we finally get

$$D_\nu(W_\nu|\phi_L|^2) - \frac{ig}{\cos\theta_W}|\phi_L|^2 Z_\mu W_\mu + (\phi_L \partial_\mu \phi_L^* - \phi_L^* \partial_\mu \phi_L)W_\mu = 0 \quad . \quad (10)$$

A similar expression was first derived by MacDowell and Trnkvist [8] for the case where ϕ_L is real. For complex ϕ_L one gets the last term in eq. (10) as an additional term.

For $\theta_W = 0$ eq. (10) reduces to

$$\partial_\nu(W_\nu|\phi_L|^2) + (\phi_L \partial_\mu \phi_L^* - \phi_L^* \partial_\mu \phi_L)W_\mu = 0 \quad . \quad (11)$$

Now let us return to the original problem in eq. (4), which we can rewrite (χ is the phase of ϕ_L)

$$\begin{aligned} &(\text{terms which vanish when } \phi_U = 0) + \\ &+ \frac{1}{2}ig \frac{e^{i\chi}}{|\phi_L|} \left[\partial_i(W_i^\dagger |\phi_L|^2) + (\phi_L^* \partial_i \phi_L - \phi_L \partial_i \phi_L^*) W_i^\dagger \right] = 0 \quad . \end{aligned} \quad (12)$$

The quantity in the square bracket vanishes because of eq. (11). Thus we can take $\phi_U = 0$ because this is consistent with the equation of motion for ϕ_U . In the following ϕ_L is denoted ϕ , and the Higgs field has the form $\begin{pmatrix} 0 \\ \phi \end{pmatrix}$.

Since we are interested in a W -dressed Z -string we take the Z -field to have the usual structure $Z_{r\theta} = -Z_{\theta r} \neq 0$, $Z_\theta \neq 0$, and all other Z fields equal to zero [1, 3]. Also, from the integrability condition (11) it is easy to see that in order to have a non-trivial solution we need two non-vanishing W -fields. Since $Z_{r\theta}$ couples to W_θ and W_r

(but not W_z and W_0) we take W_θ and W_r to be non-vanishing, and $W_z = W_0 = 0$. The W -equation (5) then gives

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial W_\theta}{\partial r} - \frac{1}{r} \frac{\partial^2 W_r}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial W_r}{\partial \theta} - \frac{W_\theta}{r^2} \\ - ig Z_{r\theta} W_r - \frac{g^2}{2} |\phi|^2 W_\theta + g^2 (W_r W_\theta^\dagger - W_\theta W_r^\dagger) W_r = 0 \quad , \quad (13) \end{aligned}$$

$$\begin{aligned} \frac{1}{r^2} \frac{\partial^2 W_r}{\partial \theta^2} - \frac{1}{r} \frac{\partial^2 W_\theta}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial W_\theta}{\partial \theta} - g^2 Z_\theta^2 W_r \\ + ig Z_{r\theta} W_\theta - \frac{g^2}{2} |\phi|^2 W_r - g^2 (W_r W_\theta^\dagger - W_\theta W_r^\dagger) W_\theta = 0 \quad . \quad (14) \end{aligned}$$

In addition to these equations it is important that we satisfy the integrability condition (11),

$$\frac{1}{r} \frac{\partial r |\phi|^2 W_r}{\partial r} + \frac{1}{r} \frac{\partial |\phi|^2 W_\theta}{\partial \theta} - 2i |\phi|^2 W_\theta \frac{m}{r} = 0 \quad (15)$$

where we used $\phi = |\phi| e^{im\theta}$, so m is the winding number for the Z -string.

The integration condition (15) incorporates features of the Z -equation (9) not incorporated in eqs. (13) and (14). Any ansatz for W_r and W_θ is therefore in danger of being in conflict with eq. (15), even if eqs. (13) and (14) are satisfied. Consistency of all three eqs. (13) - (15) is therefore a non-trivial check of any ansatz for W_r and W_θ .

We take the θ -dependence of W_r and W_θ to be

$$e^{il\theta} \quad . \quad (16)$$

Eq. (15) then gives

$$W_\theta i(l - 2m) = - \frac{1}{|\phi|^2} \frac{\partial r |\phi|^2 W_r}{\partial r} \quad . \quad (17)$$

For a Z -string $|\phi| \propto r$ for $r \rightarrow 0$. We shall now find W_θ and W_r for small values of r . Writing

$$e^{-il\theta} W_r = ar^p + \dots \quad . \quad (18)$$

with $p \geq 0$ to avoid singularities, we get from eq. (17) and $|\phi| \propto r$,

$$e^{-il\theta} W_\theta = ia \frac{p+3}{l-2m} r^p + \dots \quad . \quad (19)$$

Thus we see that because of the integrability condition the winding number enters in W_θ in the lowest order. This is not a consequence of eqs. (13) and (14).

Inserting (18) and (19) in eq. (13), we see that since $Z_{r\theta} \rightarrow \text{const}$, $Z_\theta \rightarrow \text{const} \times r$ the dominant terms are the four first terms.

Hence

$$a \left[\frac{i(p+3)}{l-2m} p^2 - ilp + il - \frac{i(p+3)}{l-2m} \right] r^{p-2} = 0 \quad ,$$

or

$$p = -2 + \sqrt{1 + l(l-2m)} \quad . \quad (20)$$

In the following we take $m = 1$. Then³

$$p = -2 + |l-1| \quad , \quad (21)$$

³If $m = 2, 3, \dots$ p is no longer an integer. If the W 's are continued to complex values of r a cut appears.

so $p \geq 0$ for $l = -1, -2, \dots$ or $l = 3, 4, \dots$. From eq. (14) we get precisely the same result (21). Thus eqs. (13) - (15) give the unique non-singular result

$$\begin{aligned} W_r &= e^{i\theta l} a r^{-2+|l-1|} + \dots, \\ W_\theta &= e^{i\theta l} \frac{ia(1+|l-1|)}{l-2} r^{-2+|l-1|} + \dots \end{aligned} \quad (22)$$

Next let us consider the energy,

$$\begin{aligned} \varepsilon &= \left| \frac{\partial W_\theta}{\partial r} - \frac{il}{r} W_r + \frac{1}{r} W_\theta - igZ_\theta W_r \right|^2 \\ &\quad + igZ_{r\theta}(W_r W_\theta^\dagger - W_\theta W_r^\dagger) + \frac{g^2}{2} |\phi|^2 (W_r W_r^\dagger + W_\theta W_\theta^\dagger) \\ &\quad - \frac{g^2}{2} (W_r W_\theta^\dagger - W_\theta W_r^\dagger)^2 + \text{other terms} \end{aligned} \quad (23)$$

where we included the kinetic energy of the W 's and the magnetic moment terms (1).

Consider now the kinetic term. From (22) its behavior is $(r^{p-1})^2$. However, it is easy to see that

$$\frac{\partial W_\theta}{\partial r} - \frac{il}{r} W_r + \frac{1}{r} W_\theta = 0 \quad \text{to order } r^{p-1} \quad (24)$$

because of (21) and (22). Also $|Z_\theta W_r|^2 \sim O(r^{2p+2})$, $|\phi|^2 |W|^2 \sim O(r^{2p+2})$. Thus, to leading order

$$\begin{aligned} \Delta &= igZ_{r\theta}(W_r W_\theta^\dagger - W_\theta W_r^\dagger) \\ &= 2ga^2 Z_{r\theta}(r=0) \frac{1+|l-1|}{l-2} r^{-4+2|l-1|} \end{aligned} \quad (25)$$

For $l = -1, -2, \dots$ we see that $\Delta < 0$. Hence, in this case the anomalous magnetic moment effect discussed previously [6]-[7] is possible.

Let us take $l = -1$. Then eq. (22) becomes

$$\begin{aligned} W_r &= e^{-i\theta} (a + O(r^2)) \\ W_\theta &= -ie^{-i\theta} (a + O(r^2)) \end{aligned} \quad (26)$$

where the order of the next-to-leading order follows from the fact that eqs. (13)-(15) are invariant under $r \rightarrow -r$ (using also the equation for ϕ and Z_0).

At first sight eq. (26) looks peculiar, since W_r and W_θ are multivalued for $r = 0$. This is, however, not a problem. First, let us notice that polar coordinates are ill-defined for $r = 0$. Therefore the components W_r and W_θ of the vector \mathbf{W} are defined relative to unit vectors in the r - and θ -directions which are themselves ill-defined for $r = 0$. Therefore at $r = 0$ it is better to consider \mathbf{W} decomposed into x - and y -components. One then finds that $W_x = a$, $W_y = -ia$. The vector \mathbf{W} is therefore perfectly single-valued for $x = y = 0$. For $r \neq 0$ it is, however, more convenient to use W_r and W_θ , so we shall continue to do this with the tacit understanding that if $r = 0$ one should transform to x - and y -coordinates.

For $p = 0$ the dominant terms in the energy are thus Δ , with $\Delta < 0$, the quartic (W^4) terms and the term $\frac{1}{2} Z_{r\theta}^2$. The $r = 0$ energy is minimized for

$$2ga^2 = Z_{r\theta}(r=0) \quad (27)$$

When this is the case we also have $F_{\mu\nu}^3(r=0) = 0$ and $D_\mu W_\nu - D_\nu W_\mu = 0$ ($r=0$). Due to the fact that $|\phi(r=0)| = 0$, we see that the full $SU(2) \otimes U_y(1)$ symmetry is restored along the axis $r=0$, and the W -fields are just pure gauge fields along this axis. Thus, one can say that the minimum energy ($r=0$) W -dressed electroweak string interpolates between two different vacua: The $r=0$ $SU(2) \otimes U_y(1)$ symmetric vacuum and the $r=\infty$ broken vacuum. For this solution the constant a is determined by $Z_{r\theta}$ ($r=0$) according to eq. (27). When we take $r > 0$ the energy becomes more complicated, with contributions from the Higgs field also. A detailed study of the energy can only be done numerically.

The quantity $Z_{r\theta}(r=0)$ is in principle determined by the quantization condition for the Z -string,

$$\int d^2x Z_{12}(x) = \frac{4\pi}{g} \quad , \quad (28)$$

which follows from the asymptotic behavior $Z_\theta \rightarrow 2/gr$ for $r \rightarrow \infty$. A precise determination requires numerical calculations. However, we have computed W_r, W_θ, Z_θ , and ϕ in the first two orders in an expansion around $r=0$. For $l=-1$ we obtain from eqs. (13) – (15)

$$\begin{aligned} e^{i\theta} W_r &= a + \frac{1}{2}(g^2 a^2 - f - \frac{1}{2}gZ_0)ar^2 + \dots \quad , \\ e^{i\theta} W_\theta &= -ia - \frac{i}{6}(5g^2 a^2 - f - \frac{5}{2}gZ_0)ar^2 + \dots \quad , \end{aligned} \quad (29)$$

where $Z_0 = Z_{r\theta}(r=0)$, and where

$$|\phi|^2 = \phi_0'^2 r^2 (1 + fr^2) \quad . \quad (30)$$

Again it is remarkable that the *three* eqs. (13)–(15) produce a consistent answer for the *two* coefficients multiplying r^2 in eqs. (29).

We have also computed the Z -field. From eqs. (9) and (29) we obtain

$$\begin{aligned} Z_\theta &= \frac{1}{2}Z_0 r + \gamma r^3 + \dots \quad , \\ \gamma &= -\frac{1}{8}g\phi_0'^2 - \frac{1}{24}ga^2(-4g^2 a^2 + 8f + 5gZ_0) \quad . \end{aligned} \quad (31)$$

Finally we studied the equation for the Higgs field,

$$\begin{aligned} -\frac{1}{r} \frac{d}{dr} r \frac{d}{dr} |\phi| &+ \left(\frac{1}{r} - \frac{g}{2} Z_\theta \right)^2 |\phi| - 2\lambda\phi_0^2 |\phi| + \\ &+ \frac{1}{2}g^2 W_i W_i^\dagger |\phi| + 2\lambda|\phi|^3 = 0 \quad . \end{aligned} \quad (32)$$

The constant f in eq. (30) is given by

$$f = -\frac{1}{9}gZ_0 - \frac{4}{9}(\lambda\phi_0^2 - \frac{1}{2}g^2 a^2) \quad . \quad (33)$$

From eq. (31) we see that to order r^3 the effect of W -condensation is to increase γ by the quantity $+\frac{1}{6}g^3 a^4$. Thus the Z -field is enhanced. This is an effect of the anti-screening property of the electroweak vacuum discussed in ref. (7).

For $r \rightarrow \infty$ the asymptotic behaviors are

$$\begin{aligned} W_r, W_\theta &\sim e^{-m_v r} \quad , \quad m_v^2 = \frac{1}{2} g^2 \phi_0^2 \quad , \\ \phi_0 - |\phi| &\sim e^{-m_H r} \quad , \\ Z_\theta - \frac{2}{gr} &\sim e^{-m_v r} \quad . \end{aligned} \tag{34}$$

Due to eqs. (32) and (34) the W 's have very little influence on $\phi_0 - |\phi|$ for $r \rightarrow \infty$.

The conclusion is thus that it appears likely that there exists a W -dressed electroweak string for $\theta_W = 0$, the consistency of which is closely related to the integrability condition (15). In principle, one should study the stability of this object by considering small perturbations. However, we think it is likely that the string is metastable, since the phenomenon of W -condensation originally was found by a stability analysis [9] showing that it pays to have W -fields for large external fields, and since a similar analysis was later performed explicitly for the electroweak string by Perkins [6].

This does, of course, not imply that the W -dressed electroweak string is absolutely stable. For example, adding Chern-Simons terms Poppitz has shown [10] that W -condensation leads to a string with a periodic structure in the z -direction if one has a finite fermion density. Such a string could also lead to baryon asymmetry.

We mention that even if one has more than one Higgs multiplet [5], it is in any case possible to lower the energy by W -condensation, as long as one has terms of the type (1) in the energy.

Finally we mention that one can see from the equations of motion that for $\theta_W \neq 0$ a magnetic field $F_{12} = \partial_1 A_2 - \partial_2 A_1$ is necessary in the string core if a W -dressed Z -string exists.

I thank Tanmay Vachaspati for many interesting discussions.

References

- [1] T. Vachaspati, Phys. Rev. Lett. **68** (1992) 1977.
- [2] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264;
A. Salam, in “Elementary Particle Theory”, ed. by N. Svartholm (Almqvist & Wiksell, Stockholm, 1968), p. 367.
- [3] H.B. Nielsen and P. Olesen, Nucl. Phys. **B61** (1973) 45.
- [4] M. James, L. Perivolaropoulos and T. Vachaspati, Phys. Rev. **D46** (1992) 5232;
M. James, L. Perivolaropoulos and T. Vachaspati, Nucl. Phys. **B395** (1993) 524.
- [5] G. Dvali and G. Senjanovic, Preprint IC/93/63 (1993);
L. Perivolaropoulos, Harvard preprint (1993).
- [6] W.B. Perkins, Phys. Rev. **D47** (1993) 5224.
- [7] J. Ambjrn and P. Olesen, Nucl. Phys. **B315** (1989) 606;
J. Ambjrn and P. Olesen, Nucl. Phys. **B330** (1990) 193;
J. Ambjrn and P. Olesen, Int. J. Mod. Phys. **A5** (1990) 4525.
- [8] S.W. MacDowell and O. Trnkvist, Phys. Rev. **D45** (1992) 3833.
- [9] N.K. Nielsen and P. Olesen, Nucl. Phys. **B144** (1978) 376.
- [10] E.R. Poppitz, Phys. Lett. **B309** (1993) 114.